Neuro/PBIO 545: Quantitative methods in neuroscience

Organization (or …)
- Lectures
  - lecture #1 introduction to mathematical topic
  - lecture #2 clean up (make sure you look at tutorial prior to lecture #2!)
- Discussion
  - journal clubs run by you (sign up for topics by Sunday evening)
  - class discussion

Topics
- weeks 1/2: Linear algebra (Rieke)
- weeks 3/4: Stochastic processes (Bair)
- weeks 5/6: Fourier analysis (Rieke)
- weeks 7/8: Differential equations (Fairhall)
- weeks 9/10: Principle components analysis (Bair)

Course website
http://rieke-server.physiol.washington.edu/People/Fred/Classes/545/545.html
(linked from PBIO site or google “pbio 545”)
Neuro/PBIO 545: Quantitative methods in neuroscience

Computers and Matlab
• We will assume working knowledge of Matlab (but not expertise!)
• Matlab tutorials (Mathworks site)
  
  *Let us know if Matlab access is a problem*

Grades (based on effort, not prior knowledge)
• Project (due last week of class + oral presentation during Finals week, 25%)
• Paper presentation during journal club (25%)
• Participation in class discussions (25%)
• Tutorial problem sets (25%; due Thurs)

Keys to class working
• slow us down with questions
• do tutorials early and *send questions by Wed/Sun night*
• read papers + participate in discussion of them
• have realistic expectations (we will try to)!
Projects

- Due during finals week
- This should be an extension or application of one of the mathematical concepts we cover in class. Start thinking right away about possibilities; we will mention them as they come up during class as well
- Submit a brief written project proposal no later than Mon Feb 27
- Discuss the proposal with one of us and get approval by Fri March 3
- Midway through the course we will have you start briefly presenting your project plans in class to help the project selection process
- For more details on project see “some logistics” link on course website
Presentations (groups of ~4):
Paper chosen to provide clear example in which a given mathematical concept has advanced our understanding of neuroscience. Presentations should include:
• Background/methods including reminder of relevant math
• Results from primary paper, focus on how mathematical concept was used to advance science
• Class discussion (non-presenting students should turn in written answer to one of a set of discussion questions)

Timeline:
• meet as a group with lecturer by 3-4 days before presentation to start to organize
• arrange additional meetings with lecturer as needed to clarify any issues that come up
• circulate list of Discussion questions within group and finalize to send out to class at least one day before presentation
What you need to do by this Sunday:

• Complete Linear Algebra Tutorial, answer questions within tutorial and turn in to Abhishek before class Mob
• Choose a topic to present and email a ranked list of 3 by Sunday (rieke@uw.edu), include a sentence about your math background. Bonus points for choosing linear algebra!
• Send me questions about items you would like clarified from lecture today or from the tutorial by midnight Sun (sooner is ok) (this will count towards your “discussion” grade!)
COORDINATE SYSTEMS IN MOTOR CONTROL

- choice of appropriate coordinate system
- conversion from one to another
SIGNALS IN TIME AND FREQUENCY DOMAIN

\[ A(t) = \sum_{n=1}^{3} A_n \sin(2\pi f_n t + \phi_n) \]

general point: computations can take simple form in some coordinate systems
GOALS FOR THIS WEEK

• Why is linear algebra useful?
  - practical analysis tools
  - visualize data/computations

• Matrix transformations and their properties

• Eigensystem and ‘natural’ coordinates
COLOR VISION AND CONE SPACE

how do we predict where input signals live in cone space?

e.g. 590 nm light or mixture of 530 nm and 620 nm light?

Hofer et al., 2005
How do we convert gun sensitivities to cone weights?

\[(R_b, R_g, R_r) \rightarrow (R_S, R_M, R_L)\]
MONITOR AND CONE SPACES

• **KEY**: monitor space and cone space describe same physical stimuli using different axes (i.e. axes “span” the same space)
• Cone space natural choice for color vision, but we control stimuli in monitor space.
• *Determine cone excitations for given input stimulus.*
• *Generate stimuli that excite only one cone type.*

need to learn how to transform from monitor to cone space and vice-versa (matrix P)
determine sensitivity of each cone to each gun, and
turn problem into set of 3 linear equations and 3 unknowns
How do we compute effect of, e.g., green monitor gun on L cone?
**MONITOR GUNS IN CONE SPACE**

- Monitor guns define directions in cone space (unit vectors) - i.e. they provide a (nonorthogonal) coordinate system.
- They span cone space if they point in different directions.

Decompose each gun into components along cone axes - relative sensitivities of cones to gun (dot product and projection).
CONVERTING FROM GUN COORDINATES TO CONE COORDINATES

Each column of $P$ is vector for that gun, expressed as cone weights - columns of $P$ provide coordinate system.

$$
\begin{bmatrix}
R_S \\
R_M \\
R_L
\end{bmatrix} =
\begin{bmatrix}
P_{bS} & P_{gS} & P_{rS} \\
P_{bM} & P_{gM} & P_{rM} \\
P_{bL} & P_{gL} & P_{rL}
\end{bmatrix}
\times
\begin{bmatrix}
R_b \\
R_g \\
R_r
\end{bmatrix}
$$

$$
\mathbf{R}_{\text{cones}} = P \mathbf{R}_{\text{guns}}
$$
Questions thus far?
MONITOR AND CONE SPACES

• Cone space natural choice for color vision, but control stimuli in monitor space.
• Determine cone excitations for given input stimulus.
• Generate stimuli that excite only one cone type.
CONE ISOLATING STIMULI AND MATRIX INVERSE

L cone isolating stimulus

\[
\begin{pmatrix}
0 \\
0 \\
1
\end{pmatrix} =
\begin{bmatrix}
P_{bS} & P_{gS} & P_{rS} \\
P_{bM} & P_{gM} & P_{rM} \\
P_{bL} & P_{gL} & P_{rL}
\end{bmatrix}
\times
\begin{pmatrix}
w_b \\
w_g \\
w_r
\end{pmatrix}
\]

• choose \((w_b \, w_g \, w_r)\)

• inverse:
\[PP^{-1} = I\]

• identity matrix \(I\):
\[I \, u = u \text{ for any } u\]

\[
\begin{pmatrix}
P_{bS} & P_{gS} & P_{rS} \\
P_{bM} & P_{gM} & P_{rM} \\
P_{bL} & P_{gL} & P_{rL}
\end{pmatrix}^{-1}
\times
\begin{pmatrix}
0 \\
0 \\
1
\end{pmatrix} =
\begin{pmatrix}
w_b \\
w_g \\
w_r
\end{pmatrix}
\]
GOALS FOR THIS WEEK

• Why is linear algebra useful?
  - practical analysis tools
  - visualize data/computations

• Matrix transformations and their properties

• Eigensystem and ‘natural’ coordinates

\[ \mathbf{v} = \mathbf{M} \times \mathbf{u} \]
TWO IMPORTANT INVERTIBLE MATRIX TRANSFORMATIONS

rotation

\[ M = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \]

scaling

\[ M = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \]

\[ v = M u \]

e.g. converting from body centered to world centered coordinates in motor systems

e.g. change in gain of one input to a cell while other remains fixed (adaptation)

KEY: invertible matrix transformations are 1-1 (i.e. every \( u \) mapped to unique \( v \), hence each \( v \) can be mapped back to \( u \))

see tutorial for more details/examples
NON-INVERTIBLE MATRIX TRANFORMATIONS

projection

\[ M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

\[ M \times \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a \\ 0 \\ c \end{pmatrix} \]

for invertible matrix, each column must be independent (n equations and n unknowns)

\[
\text{rank} = \# \text{ independent columns}
\]

(else cannot satisfy \( M^{-1}Mu = u \))
Columns of P provide coordinate system - i.e. they are monitor guns represented in cone space coordinates.

\[
\begin{bmatrix}
R_S \\
R_M \\
R_L
\end{bmatrix}
= \begin{bmatrix}
P_{bS} & P_{gS} & P_{rS} \\
P_{bM} & P_{gM} & P_{rM} \\
P_{bL} & P_{gL} & P_{rL}
\end{bmatrix}
\times
\begin{bmatrix}
R_b \\
R_g \\
R_r
\end{bmatrix}
\]
Column and Null Spaces

Null space

Column space:
-space spanned by matrix columns

Null space:
-space spanned by vectors $\mathbf{u}$ that satisfy

$$M \mathbf{u} = 0$$
COLUMN AND NULL SPACES

$$M = \begin{bmatrix}
3 & -2 & 1 \\
2 & 1 & 3 \\
-1 & 2 & 1 \\
\end{bmatrix}$$

- convergence from high dimensional space to lower dimensional space (e.g. real world images)
- color blindness
Questions thus far?